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Physics @ Null Boundaries, Edition 2022

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Abstract

To formulate gravity in spacetimes bounded by a null boundary, an arbitrary hypothetical null surface, boundary degrees of freedom (d.o.f) should be added to account for the d.o.f and dynamics in the spacetime regions excised behind the null boundary. In the D dimensional example, boundary d.o.f are labelled by D charges defined at $D-2$ dimensional spacelike slices at the null boundary. While boundary modes can have their own boundary dynamics, their interaction with the bulk modes is governed by flux-balance equations which may be interpreted as a diffusion equation describing "dissolution" of bulk gravitons into the boundary. From boundary viewpoint, boundary d.o.f obey local thermodynamical equations at the boundary. Our description suggests a new "semiclassical" quantization of the system in which boundary d.o.f are quantized while bulk is classical. This semiclassical treatment may be relevant to questions in black hole physics.

We typically face formulating physics problems in some specified regions of spacetime. The boundary which is a codimension one surface in D dimensional spacetime may have null, timelike or spacelike sections. Boundaries may be hypothetical regions in spacetime or physical surfaces; they may be at asymptotic regions of spacetime where spacetime is naturally limited to one side of the boundary or may be hypersurfaces dividing the spacetime into "inside and outside" or "front and behind" regions. In the latter case one may excise the region behind the boundary and try to formulate the problem in this excised spacetime. In this essay we describe physics from the viewpoint of the "front observer" who does not have access to the behind region. This is essentially an update on our previous "Horizon 2020" essay [1], which itself was a continuation of [2].

Among different choices for the boundary we consider a null boundary $\mathcal{N}$, denoted by $r = 0$ in Fig. 1. Any accelerated observer finds such a null boundary. This choice is also motivated by the questions regarding black holes, where the boundary models the black hole horizon. The null boundary is special as it only allows for a one-way passage of the null rays to the behind ($r < 0$) region. $\mathcal{N}$ is a null surface which is topologically $\mathbb{R}_v \ltimes \mathcal{N}_v$. In what follows we view v as the "time" coordinate for the boundary observers, $\mathcal{D}_v$ denotes the covariant time derivative along $\mathcal{N}$ and x^i span $\mathcal{N}_v$. Being a null surface, the metric on $\mathcal{N}$ is degenerate. We should stress that any two points $(v_1, x^i_1), (v_2, x^i_2)$ on $\mathcal{N}$ are out of relativistic causal contact, unless $x^i_1 = x^i_2$. So, information on these points can't be connected by a causal dynamics and the theory on $\mathcal{N}$ does not have a relativistic description. Since $\mathcal{N}$ can be locally obtained as speed of light to zero limit of a $D-1$ Minkowski space, this should be a Carrollian local field theory description [3–6], and references therein.

Choosing the null boundary $\mathcal{N}$ as described above, partially fixes D dimensional diffeomorphisms to $D-1$ diffeomorphisms on $\mathcal{N}$ plus local scaling of the r coordinate, $r \to W(v, x^i)r$. There are hence D "residual diffeomorphisms", respectively corresponding to local translations in v and x^i plus $W(v, x^i)$. Since ∂_r is a null direction, W generates local boosts on $\mathcal{N}$. Boosts along x^i directions do not keep $\mathcal{N}$ null and are not among our symmetry generators. Therefore, the boundary theory is expected to respect "$D-1$ dimensional conformal Carrollian" symmetry. In this essay we focus on the physical picture emerging from recent papers [7–10] and in particular [11, 12], without delving into interesting technicalities of the analyses. For a detailed analysis one may look at those papers.

Front observers, observers in $r > 0$ region, may see things falling in, but not coming out. We are going to excise $r < 0$ region and only focus on $r \geq 0$ region. Front observers interpret an infalling flux as something "dissolving" into the null boundary. In order this picture to physically make sense one should add appropriate boundary degrees of freedom (b.d.o.f) which reside on $\mathcal{N}$ and readjust themselves as a response to the dissolution of the flux. This readjustment is governed by the flux-balance equations which are simply (Einstein) field equations projected along and computed at $\mathcal{N}$, the Raychaudhuri and Damour equations at $\mathcal{N}$. There are $D-1$ such equations. One should note that while the details of these equations do depend on the gravity theory we are considering, their existence and that they are just first order differential equations in time v, is merely a consequence of diffeomorphism invariance of the theory and do not depend on the theory.

One may construct space of all solutions to Einstein GR with $\mathcal{N}$ as the null boundary through a perturbative expansion in r. This solution space is specified by D functions over $\mathcal{N}$, namely D arbitrary

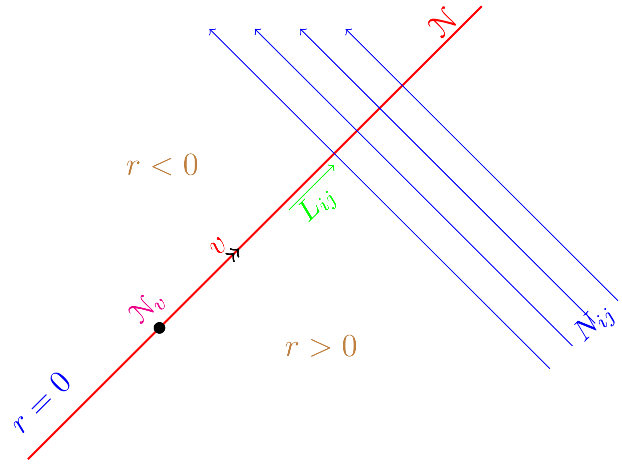

Figure 1: $\mathcal{N}$ is a null boundary at $r = 0$. v is the null coordinate along $\mathcal{N}$ and the $D-2$ dimensional "transverse" space $\mathcal{N}_v$, constant v surfaces on $\mathcal{N}$, is spanned by coordinates $x^i, i = 1, 2, \cdots, D-2$. The null boundary $\mathcal{N}$ does not necessarily have an initial or endpoint. We excise the $r < 0$ region and formulate physics in $r \geq 0$. N_{ij}, also called (Bondi) news, parameterize infalling null rays while L_{ij} are null rays propagating along the boundary. The passage of N_{ij} through $\mathcal{N}$ is interpreted as dissolution of gravitons onto the boundary from the viewpoint of observers in $r \geq 0$ region.

functions of v and transverse coordinates x^i, plus the bulk graviton modes which can propagate in the bulk. The D b.d.o.f may be labeled by the set of D charges $\mathcal{Q}_A(v, x^i), A = 1, 2, \cdots, D$, associated with the D residual diffeomorphisms described above. $\mathcal{Q}_A$ consist of two "scalar" modes $\Omega(v, x^i), \mathcal{P}(v, x^i)$ and a "vector" mode $\mathcal{J}_i(v, x^i)$. The graviton modes fall into two classes, parametrized by symmetric traceless tensors $N_{ij} = N_{ij}(v, x^i), L_{ij} = L_{ij}(v, x^i)$, *cf.* Fig. 1. $\Omega(v, x^i) = \sqrt{\det \Omega_{ij}}$, is the charge associated with the local boosts at $\mathcal{N}$ and $\Omega_{ij}(v, x^i)$ is the metric on codimension two surface $\mathcal{N}_v$. N_{ij} is the trace-free part of $\mathcal{D}_v \Omega_{ij}$ and the flux-balance equations involve first order v derivatives of the boundary modes and N_{ij}, and not L_{ij}. These equations from the boundary observer viewpoint are like an ordinary diffusion equation, describing how the news N_{ij} dissolves/diffuses as it reaches the boundary. The same equations can be interpreted as "null boundary memory effect" as they tell us how the news N_{ij} is encoded into the b.d.o.f after its dissolution. The boundary memory is a local effect on $\mathcal{N}_v$, while it involves an integration over v. See [11] for the details of analysis.

Solution space is a phase space equipped with a symplectic two-form $\boldsymbol{\Omega}$:

$$\boldsymbol{\Omega} = \frac{1}{16\pi G} \int_{\mathcal{N}} \sum_{A=1}^{D} \delta \mathcal{Q}_A \wedge \delta \mu^A + \delta(\Omega N_{ij}) \wedge \delta \Omega^{ij}. \tag{1}$$

where G is the Newton constant, $\delta X, \forall X$ is a one-form over the solution space and Ω^{ij} is inverse of metric Ω_{ij}. , $\mu^A = \mu^A(v, x^i)$ are canonical conjugates to the charges $\mathcal{Q}_A$ and are related to $\mathcal{Q}_A$ and the graviton modes N_{ij} through the balance equations. The canonical conjugate to $\mathcal{P}$ is $\mathcal{D}_v \Omega = \Omega(v, x^i)\Theta(v, x^i)$ where Θ is the expansion of the null surface, the canonical conjugate to Ω is local acceleration of null rays generating $\mathcal{N}$ and canonical conjugate to $\mathcal{J}_i$ are angular velocity of the same null rays.

The above description of the solution space, especially noting that Ω is the charge associated to boosts on $\mathcal{N}$ and its canonical conjugate variable is local acceleration, suggests that there should be a thermodynamical interpretation. In this thermodynamical description, *entropy density* at any constant v on $\mathcal{N}$ is $4G\Omega$, extending seminal Wald's result [13, 14], and its conjugate variable is the *local temperature* (times 4π), extending seminal Unruh's analysis [15]. The other terms, too, have a natural thermodynamic description, with local first law, local Gibbs-Duhem and local zeroth law, as established in [12]. Here by local we mean local on $\mathcal{N}$. This is in general an open thermodynamical system as it can be out of (local) equilibrium due to the passage of news N_{ij} or having a non-zero expansion Θ; thermal equilibrium may be achieved only in the absence of news [12], when the boundary theory becomes a closed (isolated) thermodynamical system. We stress that balance equation which is describing the rearrangement of b.d.o.f due to the passage of N_{ij} through $\mathcal{N}$, should not be viewed as a (relativistic) dynamical equation. This rearrangement happens locally (instantaneously) at any given v to ensure diffeomorphism invariance of the D dimensional theory.

Analyses in [12] revealed another interesting fact: Requiring the zeroth law, when the boundary system is closed, yields canonical Poisson brackets in which $\Omega, \mathcal{P}$ form a Heisenberg algebra,

$$\{\Omega(v, x^i), \mathcal{P}(v, y^i)\} = \frac{1}{4G}\delta^{D-2}(x - y), \tag{2}$$

and the Poisson bracket $\{\mathcal{J}_i(v, x^i), \mathcal{J}_j(v, y^i)\}$ takes the form of the algebra of $D-2$ dimensional diffeomorphisms for any v. That these Poisson brackets have the same form for any given v is a manifestation of the fact the b.d.o.f can be defined at any given v, on the codimension two surface $\mathcal{N}_v$; explicitly, the d.o.f of the boundary theory are defined on corners, resonating the viewpoint advocated in some recent papers [16, 17]. As argued, the b.d.o.f can be governed by a well defined dynamics in v which cannot be a relativistic one, it should be a Carrollian evolution. This dynamics, however, is not specified through our analysis here and is free to be chosen.

To summarize, for any locally accelerated observer we need to formulate physics on one side of a null surface. This system is an open thermodynamic system; the dissolution of bulk infalling modes into this system is governed by the flux-balance equations. The configuration/phase space of the system is a direct sum of boundary and bulk modes. The boundary d.o.f may be parametrized by the area density Ω at a given v and its canonical conjugate variable is $\mathcal{P}$. This description is suggestive of a new "semiclassical" description of the system where the boundary mode is treated quantum mechanically while the bulk mode N_{ij} is kept classical. This semiclassical description may be relevant to questions regarding black hole microstates and the information puzzle.

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